Simulation and Modeling of Turbulent Jet Noise

T. Colonius, A. Sinha, D. Rodríguez, A. Towne, J. Liu, G.A. Brès, D. Appelö and T. Hagstrom

1 Introduction

Jet noise reduction remains an important long-range goal in commercial and military aviation. Compared with their early counterparts, modern, ultrahigh-bypass-ratio turbofans on commercial aircraft are very quiet, but ever more stringent noise regulations dictate further reductions. In addition, hearing loss by personnel and community noise issues are prompting the military to seek noise reduction on future tactical aircraft. Further increase in bypass ratio not being a practical option, military applications in particular call for nuanced approaches to noise reduction including mixing devices like chevrons or even active noise control approaches using unsteady air injection. In this paper, we briefly review some recent developments in theoretical, experimental and computational approaches to understanding the sound radiated by large-scale, coherent structures in jet turbulence that might guide these noise reduction efforts.

T. Colonius (✉) · A. Sinha · D. Rodríguez · A. Towne · J. Liu
Mechanical Engineering, California Institute of Technology, Pasadena, CA, USA
e-mail: colonius@caltech.edu

G.A. Brès
Cascade Technologies Inc., Palo Alto, CA, USA
e-mail: gbres@cascadetechnologies.com

D. Appelö
Applied Mathematics, University of New Mexico, Albuquerque, NM, USA
e-mail: appelo@math.unm.edu

T. Hagstrom
Mathematics, Southern Methodist University, Dallas, TX, USA
e-mail: thagstrom@smu.edu

J. Fröhlich et al. (eds.), *Direct and Large-Eddy Simulation IX*,
ERCOFTAC Series 20, DOI 10.1007/978-3-319-14448-1_38

2 Wavepacket Models

We begin by discussing wavepackets–coherent but intermittent advecting disturbances that are correlated over distances far exceeding the integral scales of turbulence and that are, on one hand, related to fundamental low frequency instabilities (or more precisely amplified modes) of the turbulent mean flow field, and, on the other hand, directly correlated with the most intense (aft-angle), peak frequency sound radiation [1].

In particular, large eddy simulations (LES) and advanced experimental diagnostics are providing richer and higher fidelity data sets from which acoustically important motions can be educed and used to validate theories of noise emission from large-scale structures. In previous studies, we have shown in particular how the combined data from microphone arrays, time-resolved PIV data, and LES data sets have been used to validate reduced-order models for wavepackets in both subsonic [2, 3] and supersonic [4, 5] regimes. Figure 1 provides an example comparing supersonic wavepackets computed via the Parabolized Stability Equations (PSE) with those educed using proper orthogonal decomposition of LES data. Further details are presented in Ref. [6].

3 PSE and Beyond

In constructing reduced-order models to describe the evolution of wavepackets in turbulent jets, we have up to now relied on the linearized Parabolized Stability Equations (PSE) [7] to solve for disturbances to the (assumed known) mean turbulent flow. PSE is an *ad hoc* generalization of linear stability theory (LST) that captures the downstream evolution of wavepackets using a spatial marching technique in which initial perturbations are specified at the jet inlet and propagated downstream by integration of the PSE equations.

Despite their name, the PSE equations are not parabolic in the downstream direction due to the existence of upstream acoustic modes in the PSE operator [8]. These elliptic modes must be eliminated to prevent instability in the downstream march. PSE uses implicit Euler integration along with a *minimum* step size restriction to numerically damp these modes. This parabolization strategy allows a stable march, but has the unintended consequence of damping all of the Euler modes, not just the unstable upstream acoustic ones. In particular, the downstream acoustic modes are heavily damped, limiting PSEs ability to properly capture downstream acoustic radiation.

A new spatial parabolization technique has recently been developed [9] that explicitly removes the unstable upstream modes without damaging the downstream modes, resulting in a fully parabolic one-way Euler equation that can be stably marched without numerical damping. The upstream modes are removed using a recursive

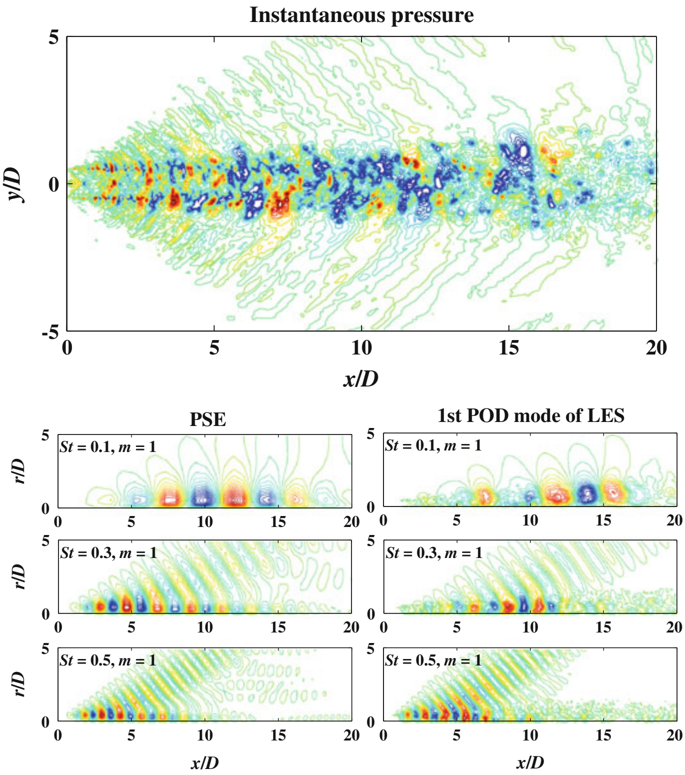

Fig. 1 *Top* instantaneous pressure contours in a plane through the nozzle exit from LES of a heated ($T_j/T_\infty = 1.74$), Mach 1.5 jet simulated with the unstructured grid compressible finite-volume solver "Charles" (see [4]). *Bottom* comparison of linear disturbances about the mean turbulent flow field computed with the PSE Ansatz to wavepackets educed from the LES data using the temporal and azimuthal Fourier transform and proper orthogonal decomposition (most energetic mode shown)

filtering technique that was originally developed for generating nonreflecting boundary conditions [10, 11]. Since the downstream acoustic modes are accurately retained, the one-way Euler equations are capable of accurately capturing downstream acoustic radiation.

This improvement over PSE is demonstrated by the results shown in Fig. 2. An LST Kelvin-Helmholtz eigenfunction is specified at the inlet of a mixing layer and propagated downstream by both PSE and the one-way Euler equations. Reasonable agreement is observed in the near-field, but the PSE solution contains little-to-no acoustic radiation while the one-way Euler solution includes a strong, directive acoustic field generated by the growth and decay of the near-field wavepacket. This method is currently being extended to turbulent jets and has the potential to provide significantly improved noise predictions, particularly for subsonic jets.

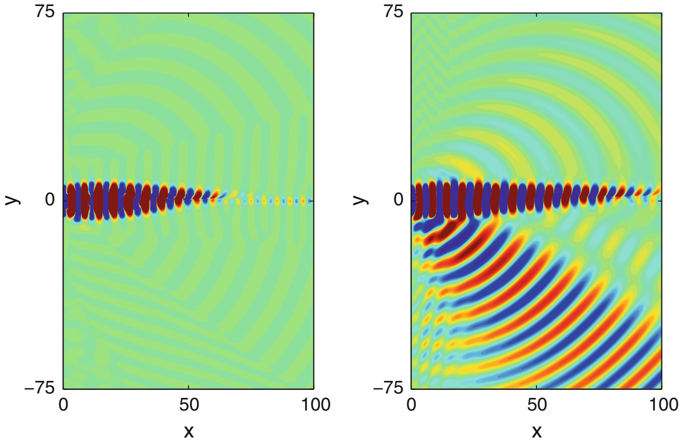

Fig. 2 Pressure field generated by a subsonic Kelvin-Helmholtz instability in a linearly spreading mixing layer with fast and slow stream Mach numbers of 0.8 and 0.2, respectively. Solution computed using **a** PSE and **b** the one-way Euler equation

4 Faster Computation

While reduced-order models are needed in the near term as surrogates for large, time-consuming simulations, in future LES will likely be used directly to provide function evaluations to formal optimization techniques. LES is being increasingly used to study jet noise in both academic and industrial settings. A range of technical issues such as numerical dispersion and dissipation, boundary conditions, extrapolation of acoustics to the far field, the inclusion of nozzle geometry and inlet disturbances, and the development of hybrid shock capturing schemes are but a few of the many technical challenges that must be resolved in obtaining high-fidelity predictions for the far acoustic field. The unstructured, locally adaptive, compressible flow solver “Charles” developed at Cascade Technologies, and used in the aforementioned wavepacket studies, leverages advances in these component algorithms to provide a unified approach towards best practices in jet noise simulation [4]. Like most modern CFD methods, Charles is designed and implemented using domain decomposition and Message Passing Interface (MPI) to exploit massively parallel, distributed memory environments.

For very large grids with $O(10^8)$ degrees of freedom, these algorithms can still achieve nearly perfect parallel scaling to $O(10^5)$ cores, but the Amdahl limit is a serious impediment to the efficiency of future jet noise computations on peta- and exa-flop machines, especially in the context of moderate-scale computations of about $O(10^7)$ grid points. Likewise, the so-called von Neumann bottleneck associated with widening speed gap between floating-point operations and memory fetches is driving research into methods that maximize the amount of operations conducted on the same data. We close this review by briefly highlighting progress on a class of methods known as Hermite methods (see [12] and references therein), which can be designed to have very high computation-to-communication ratio.

Hermite methods are arbitrary-order polynomial-based general-purpose methods for solving time dependent PDE. In one dimension the degrees of freedom at a single grid point in a Hermite method can be thought of as the $m+1$ coefficients in a degree m polynomial centered at the grid point or, equivalently, the solution and the m first spatial derivatives at that grid point. In d-dimensions this generalizes to the $(m+1)^d$ coefficients in a centered tensor product polynomial or the solution and (mixed) derivatives in the d directions up to degree m.

Advancing the solution in time is a two stage procedure. First the unique degree $(2m + 1)$ tensor product polynomial that interpolates the polynomials at the 2^d corners of a d-dimensional cuboid is constructed. This polynomial is centered at the midpoint of the cuboid and it also interpolates the solution and its derivatives at the corners of the cell, hence the name Hermite (interpolation) method. In the second stage the $(2m+2)^d$ coefficients of the Hermite interpolation polynomial are expanded in a temporal Taylor series. The coefficients in this Taylor series are found by applying a recursion relation derived by a Cauchy-Kowalewski procedure applied to the PDE at hand and the solution is advanced by evaluation the series at a later time. The procedure is repeated on the cell centered data to complete a full time step. For problems with smooth solutions this yields a method of order $2m+1$ in space and time.

For a linear PDE the main computational cost, $\mathcal{O}(m^d)$ per degree of freedom, is forming the Hermite interpolating polynomial while for a non-linear PDE with product non-linearities the main cost is computing the multiplication of degree $(2m + 1)$ tensor product polynomials needed for the coefficients in the temporal Taylor series. This cost scales as $\mathcal{O}(m^{d+1})$ if a direct polynomial multiplication algorithm is used or $\mathcal{O}(m^d \log(m))$ if a FFT based algorithm is used.

One of the best features of the Hermite method, and central to our claim to have high computation to communication ratio, is that for a wave dominated problem (high Reynolds number) the time step is limited only by the wave speeds of the problem and not by the order of the method (as is the case for discontinuous Galerkin or classic spectral elements which have to take at least m times smaller time steps.) Thus, using a Hermite method for high Reynolds number flows yields an algorithm whose error scales like $\mathcal{O}(h^{(2m+1)})$ with a cost $\mathcal{O}(m^d \log(m))$ per degree of freedom to advance the solution one time step and with a need to communicate at least m times less frequently than other polynomial spectral methods. The locality of the method is highly useful for efficient parallel implementation as well as tackling the von Neumann bottleneck.

References

1. Jordan, P., Colonius, T.: Wave packets and turbulent jet noise. Annu. Rev. Fluid Mech. **45**, 173–195 (2013)
2. Cavalieri, A.V.G., Jordan, P., Colonius, T., Gervais, Y.: Wavepackets in the velocity field of turbulent jets. AIAA Paper 2012–2115 (2012)

3. Gudmundsson, K., Colonius, T.: Instability wave models for the near field fluctuations of turbulent jets. J. Fluid Mech. **689**, 97–128 (2011)
4. Brès, G.A., Nichols, J.W., Lele, S.K., Ham, F.E.: Towards best practices for jet noise predictions with unstructured large eddy simulations. AIAA Paper 2012–2965 (2012)
5. Rodríguez, D., Sinha, A., Brès, G.A., Colonius, T.: Parabolized stability equation models in turbulent supersonic jets. AIAA Paper 2012–2117 (2012)
6. Rodríguez, D., Sinha, A., Brès, G.A., Colonius, T.: Acoustic field associated with parabolized stability equation models in turbulent jets. AIAA Paper 2013–2279 (2013)
7. Herbert, T.: Parabolized stability equations. Annu. Rev. Fluid Mech. **29**, 245–283 (1997)
8. Li, F., Malik, M.R.: On the nature of pse approximation. Theor. Comput. Fluid Dyn. **8**, 253–273 (1996)
9. Towne, A., Colonius, T.: Improved parabolization of the euler equations. AIAA Paper 2013–2171 (2013)
10. Givoli, D., Neta, B.: High-order nonreflecting boundary scheme for time- dependent waves. J. Comput. Phys. **186**, 24–46 (2003)
11. Hagstrom, T., Warburton, T.: A new auxiliary variable formulation of high-order local radiation boundary conditions: corner compatibility conditions and extensions to first-order systems. Wave Motion **39**, 890–901 (2004)
12. Appelö, D., Inkman, M., Hagstrom, T., Colonius, T.: Hermite methods for aeroacoustics: Recent progress. AIAA Paper 2011–2757 (2011)